Q Find the foci, directrices and eccentricity of the ellipse $3x^2+4y^2=12$

Soln Given ⇒ $3x^2+4y^2=12$

$$\Rightarrow \frac{\cancel{3}x^2}{\cancel{12}\,4} + \frac{\cancel{4}y^2}{\cancel{12}\,3} = 1$$

$$\Rightarrow \frac{x^4}{4}+\frac{y^2}{3}=1$$ which is in standard form.

$a^2=4,\ b^2=3$

Now since $b^2=a^2(1-e^2)$

$3=4(1-e^2) \Rightarrow 1-e^2=\frac{3}{4}$

$\Rightarrow e^2=1-\frac{3}{4}=\frac{1}{4} \Rightarrow e=\pm\frac{1}{2}$

Also we know

$ae=1$ ⇒ So the co-ordinates of foci are $(-1,0)$ and $(1,0)$

Q Find Centre and Eccentricity.

$$2x^2+3y^2-4x+5y+4=0$$

$$2(x^2-2x+1)+\underbrace{(3y^2+5y)}+4=2$$

$$\Rightarrow 2(x-1)^2+3\left(y^2+\frac{5}{3}y+\ \right)=2-4$$

$$\Rightarrow 2(x-1)^2+3\left(y^2+2\cdot\frac{5}{6}y+\frac{25}{36}\right)=-2+\frac{25}{12}$$

$$\Rightarrow 2(x-1)^2+3(y+5/6)^2=1/12$$

$$\Rightarrow \frac{(x-1)^2}{1/24}+\frac{(y+5/6)^2}{1/36}=1 \quad \text{—— } 1$$

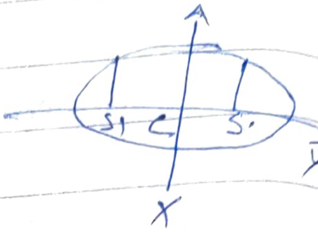
$y=a$

$\frac{5}{3}y=2ab \Rightarrow b=\frac{5}{6}$

$b^2=?\ \frac{25}{36}$

Put $x-1 = X$ and $y + 5/6 = Y$

then ① $\Rightarrow \dfrac{X^2}{1/24} + \dfrac{Y^2}{1/36} = 1$



$\underset{S_1}{(-x, y)}$, $\boxed{\underset{C}{(x, -y)}}$, $\underset{S}{(-x, -y)}$,

$\downarrow$

C is centre.

$(-1, 5/6) \to S$

$(-1, -5/6) \to S_1$

(X, Y) centre is $(1, -5/6)$

$$b^2 = a^2(1-e^2)$$

$$\Rightarrow \frac{1}{36} = \frac{1}{24}(1-e^2)$$

$$\Rightarrow (1-e^2) = \frac{24}{36} = \frac{2}{3}$$

$$\Rightarrow e^2 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow e = \pm\frac{1}{\sqrt{3}}$$

4.4. FORM OF THE CURVE

(i) Since the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ contains only even powers of x and y, the curve (ellipse) is symmetrical with regard to both the x-axis and the y-axis. For if (x, y) be a point on the curve so also are $(-x, y)$, $(x, -y)$ and $(-x, -y)$. Every line through C will thus meet the curve in two points equidistant from C. The point C is therefore called the *centre*.

(ii) The curve cuts the x-axis in two points $x = \pm a$. This is obtained by putting $y = 0$ in the equation of the ellipse. The points A and A_1, thus correspond to $x = a$ and $x = -a$ respectively. Similarly the curve cuts the y-axis in two points $y = \pm b$.

(iii) In order to know whether the curve extends beyond $x = a$, we write the equation in the form

$$y = \pm \frac{b}{a}\sqrt{a^2 - x^2}.$$

If we give to x values greater than a or less than $-a$, y is imaginary i.e. no part of the curve lies to the right of the point $(a, 0)$ or to the left of the point $(-a, 0)$. Similarly by writing the equation in the form

$$x = \pm \frac{a}{b}\sqrt{b^2 - y^2}$$

we observe that no part of the curve lies above $y = b$ or below $y = -b$.

Hence the curve lies entirely within the lines $x = \pm a$; $y = \pm b$.

That is, the ellipse is a closed curve as given in the figure of the preceding Art.

4.5. AN ELLIPSE HAS TWO FOCI AND DIRECTRICES

On the line CA take points S_1 and X_1 (see fig. of Art 4.3.) such that $CX_1 = \frac{a}{e}$ and $CS_1 = ae$.

Through X_1 draw $X_1M_1 \perp CA$.

It is not difficult to see that the ellipse which has been described with S as the focus and MX as the directrix could have been equally described with S_1 as the focus and M_1X_1 as the directrix; the eccentricity remaining the same.

Thus an ellipse has two foci and two corresponding directrices.

The co-ordinates of the two foci (S and S_1) of the ellipse are $(-ae, 0)$ and $(ae, 0)$ i.e. $(\pm\sqrt{a^2 - b^2}, 0)$ and the equation of the corresponding directrices are

$$x = -\frac{a}{e} \text{ and } x = \frac{a}{e}.$$

Cor. : *The focal chord* $SP = ePM = eXN = e(ON + OX)$

$$= e\left(x + \frac{a}{e}\right) = a + ex.$$

Similarly $S_1P = a - ex$.

4.6. AXES AND LATUS RECTUM

Definition—The two lines AA_1 and BB_1 with respect to which the curve is symmetrical are called the axes of the ellipse. As $AA_1 > BB_1$ [since $b^2 = a^2(1 - e^2)$, $b < a$] AA_1 is called the *major axis* and BB_1 the *minor axis*.

The chord through the focus parallel to the directrix is called the *latus rectum* of the ellipse.

If LSL_1 be the latus rectum, its equation is $x = -ae$.

Now the co-ordinates of the point L are $(-ae, y)$.

Since it lies on the ellipse, we have

$$\frac{a^2e^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad y^2 = b^2(1 - e^2) = b^2 \cdot \frac{b^2}{a^2}$$

$$\Rightarrow \quad y = \frac{b^2}{a}$$

i.e. $$LS = \frac{b^2}{a}.$$

Hence the length of the latus rectum

$$LSL_1 = \frac{2b^2}{a}.$$

Diameter—Any chord passing through the centre of the ellipse is called a diameter.